Vector algebra

The Maxwell equations are the set of four fundamental equations governing electromagnetism (i. e. the behavior of electric and magnetic fields).They were first written down in complete form by physicist James Clark Maxwell, who added the so-called displacement current term to the final equation.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1.1}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{displ} \tag{1.2}$$

$$\nabla \cdot \vec{D} = \rho \tag{1.3}$$

$$\nabla \cdot \vec{B} = 0 \tag{1.4}$$

$\vec{E}$ - is the field strength (or intensity)
$\vec{D}$ - is electric induction (or electric flux density, electric displacement)
$\vec{H}$ - is magnetic field strength (or intensity)
$\vec{B}$ - is magnetic induction (magnetic flux density)
$\vec{J}_{displ}$ - is the vector current density
ρ - is the density of electric charge
$\vec{D} = \varepsilon\vec{E} + \vec{P}$ - is the electric induction (electric flux density, Electric displacement), ε_0 is the permittivity of free space and $\vec{P}$ is the polarization. In linear isotropic medium.
$\vec{D} = \varepsilon\vec{E}$, where ε is a constant; $\varepsilon_0\mu_0 = c^{-2}$

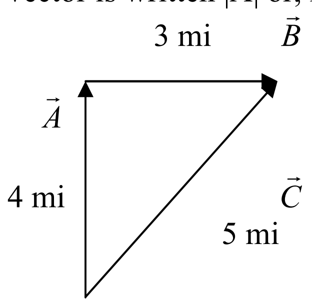
Where

∇	is a vector operator
$\nabla \cdot$	is the divergence
$\nabla \times$	is the curl

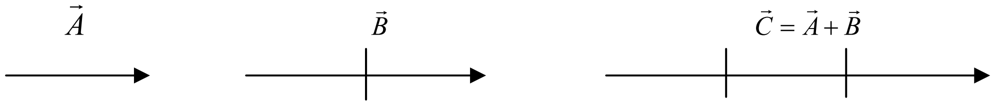
If we walk 4 miles due north and then 3 miles due east (Fig. Ex.1.1), we have gone a total of 7 miles. But we are only 5 miles (mi). The strength line going from one point to another has direction as well as magnitude (length), and it is essential to take both into account when we combine them. Such objects are called VECTOR:

Velocity, acceleration, force, momentum, are other example.

Vector quantities are represented by symbols in boldface roman type **A** or we can denote by arrow A (**A**, $\vec{A}$, $\underset{\leftarrow}{A}$).
The magnitude of a vector is written |A| or, more simply **A.**

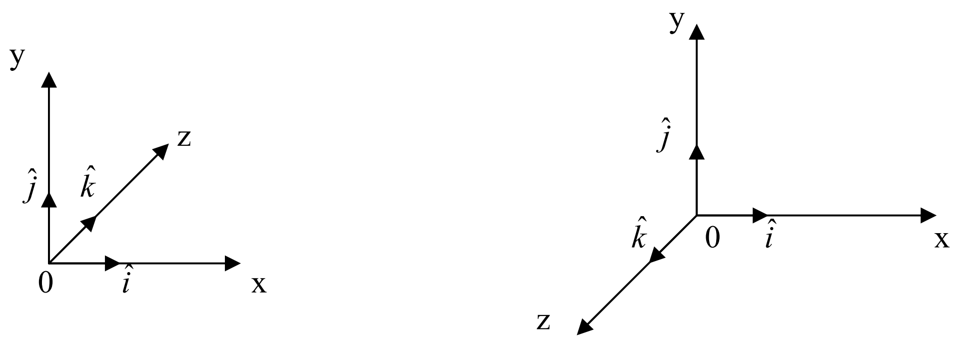


By contrast, quantities that have magnitude but no direction are called scalars:
mass, charge, density and temperature. El. Field potential φ



Vector $\vec{A}$ and $\vec{B}$ are directed eastward, with the magnitude of $\vec{B}$ being twice of $\vec{A}$. (We can say, the vector $\vec{C}$ has a magnitude three times that of $\vec{A}$)

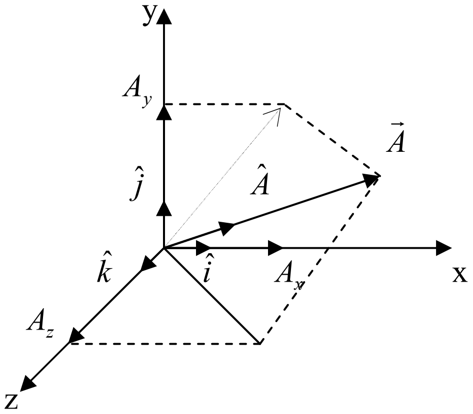
Since vector may have in general an arbitrary orientation in three dimensions, we need to define a set of three reference directions at every point in space in terms of which we can describe vector drawn at the point. It is convenient to choose these reference directions to be mutually orthogonal. Two combinations are possible for the orientations of the coordinate system



Left-Hand Right-Hand

Fig. Ex.1.1 Set of three orthogonal unit vectors in a right-handed system.
$\hat{i}$, $\hat{j}$, $\hat{k}$ are the coordinate vectors, the basis vectors, unit vectors.

This means that whey each have a length (or magnitude) of exactly 1. Usually we write unit vectors with a “hat” over them instead of an arrow.
There is one basis vector for each direction in which we can move. In three dimensions, this gives us three basis vectors, one along each coordinate axis.
The basis vector that points in the direction of the +x is called $\hat{i}$, the vector pointing toward the +y axis is called $\hat{j}$ and the vector in the z direction is called $\hat{k}$. $\hat{i} = (1,0,0); \hat{j} = (0,1,0), \hat{k} = (0,0,1)$
We note that the magnitude of $(4\hat{i} + 6\hat{j})$ is $\sqrt{4^2 + 6^2} = 7.211$. The magnitude of vector $\vec{A}$ is $(4\hat{i} + 6\hat{j} - 2\hat{i}) = \sqrt{4^2 + 6^2 + 2^2} = 7.483$



A_x, A_y, A_z are called components of $\vec{A}$

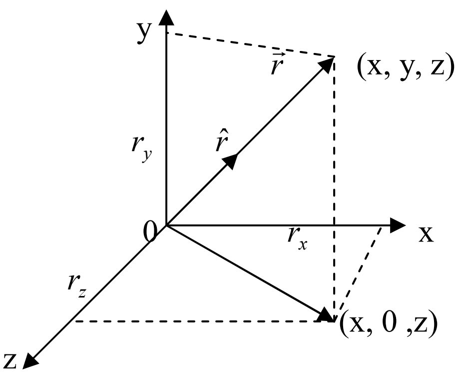
An arbitrary vector $\vec{A}$ can be expanded in terms of these basis vectors (Fig. Ex1.2, Ex.1.3)

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} = (A_x, A_y, A_k)$$

$$\left|\vec{A}\right| = A = \sqrt{A_x{}^2 + A_y^2 + A_z^2}$$

$$\vec{A} = \hat{A}A$$

$$\hat{A} = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{\vec{A}}{A}$$



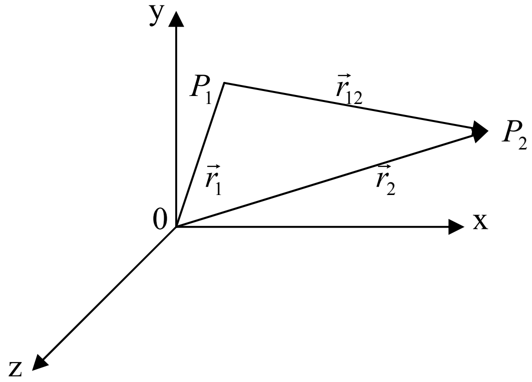
$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} = (r_x, r_y, r_z)$$

$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\underline{\hat{A}} = \hat{\vec{A}} = \mathrm{Re}(\hat{\vec{A}}) + iJm(\vec{A}) = \vec{A}' + i\vec{A}''$$

$$\underline{\vec{E}} = \vec{E}' + i\vec{E}'', \; i = \sqrt{-1}$$



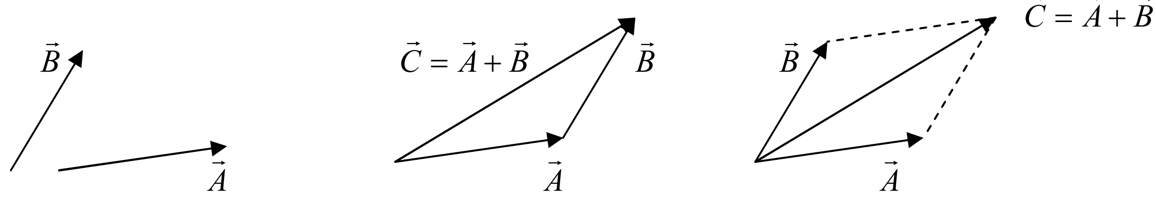
$P_1(x_1, y_1, z_1)$
$P_2(x_2, y_2, z_2)$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$
$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

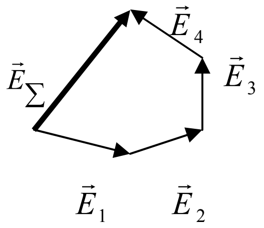
$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$|\vec{r}_{12}| = r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Vector Addition

A vector is a mathematical object that has a magnitude and a direction.
Addition of two vectors is accomplished by laying the vectors head to tail in sequence to create a triangle such as shown in figure.



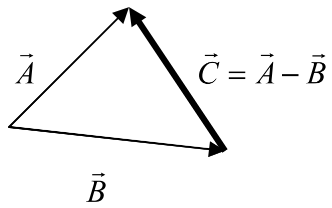
$$\vec{A} + \vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z) + (\hat{i}B_x + \hat{j}B_y + \hat{k}B_z) =$$
$$(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) =$$
$$(A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

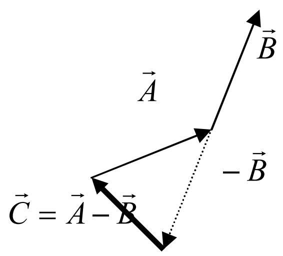


$$\vec{E}_\Sigma = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

Subtraction is one of the four basic arithmetic operations.

$\vec{C} = \vec{A} - \vec{B}$

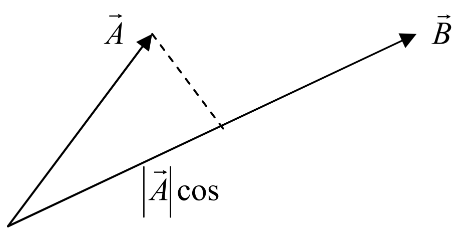




The dot product=the scalar product and The cross product=the vector product

$\vec{A} \cdot \vec{B}$ $\qquad$ $\vec{A} \times \vec{B}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos \vartheta = AB \cos \theta$$
$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$$
$$\hat{i} \cdot \hat{j} = ... = 0$$



The projection of $\vec{A}$ onto $\vec{B}$

Example 1.1

$\vec{a} = 1\hat{i} + 3\hat{j} + 2\hat{k} = (1,3,-2) = \langle 1,3,-2 \rangle$

$\hat{b} = \langle -2, 4, -1 \rangle$