CMR.1-1. CIRCULAR CAVITY RESONATOR

A circular cylindrical resonator can be formed from a circular cylindrical waveguide section and conducting walls at both ends of a waveguide. Resonator sizes are: a radius R and a length l (Fig. **CMR.1)**

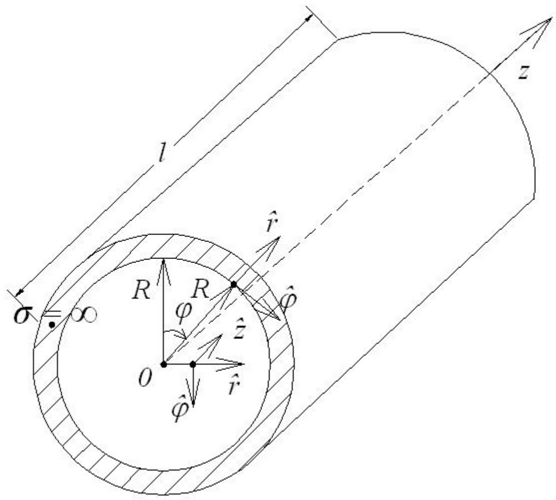

Fig. CMR.1 A cylindrical cavity resonator

EXPRESSIONS FOR THE FIELD COMPONENTS OF THE ELECTRIC FIELD IN A RESONATOR, CREATED ON THE BASE OF A CIRCULAR WAVEGUIDE.

Fedorov, p. 164:

Electric waves E_{mnp} (TH_{mnp}):

$$\dot{E}_{z0} = C_1 J_m\left(\frac{\nu_{mn}}{R} r\right)\cos(m\varphi), \quad \text{(CMW.1)}$$

$$(k_\perp)_{mn} = k_{\perp mn} = \frac{\nu_{mn}}{R} \quad \text{(CMW.2)}$$

$$\dot{E}_r^{in} = -i\frac{h}{k_\perp^2}\frac{\partial \dot{E}_{z0}}{\partial r}\exp(-ihz) \quad \text{(CMW.3)}$$

$$\dot{E}_\varphi^{in} = -i\frac{h}{k_\perp^2 r}\frac{\partial \dot{E}_{z0}}{\partial \varphi}\exp(-ihz) \quad \text{(CMW.4)}$$

$$\dot{E}_z^{in} = \dot{E}_{z0}\exp(-ihz) \quad \text{(CMW.5)}$$

$$\dot{H}_r^{in} = i\frac{\omega\varepsilon}{k_\perp^2 r}\frac{\partial \dot{E}_{z0}}{\partial \varphi}\exp(-ihz) \quad \text{(CMW.6)}$$

$$\dot{H}_{\varphi}^{in} = -i\frac{\omega\varepsilon}{k_{\perp}^{2}}\frac{\partial \dot{E}_{z0}}{\partial r}\exp(-ihz) \qquad \text{(CMW.7)}$$

$$\dot{H}_{z}^{in} = 0 \qquad \text{(CMW.8)}$$

Using (CMW.1) and (CMW.2) we obtain:

$$\dot{E}_{r}^{in} = -i\frac{hR}{\nu_{mn}}C_1 J_m'\left(\frac{\nu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \qquad \text{(CMW.9)}$$

$$\dot{E}_{\varphi}^{in} = i\frac{hR}{\nu_{mn}^{2}r}C_1 m J_m\left(\frac{\nu_{mn}}{R}r\right)\sin(m\varphi)\exp(-ihz) \qquad \text{(CMW.10)}$$

$$\dot{E}_{z}^{in} = C_1 J_m\left(\frac{\nu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \qquad \text{(CMW.11)}$$

$$\dot{H}_{r}^{in} = -i\frac{\omega\varepsilon R^2}{\nu_{mn}^{2}r}C_1 m J_m\left(\frac{\nu_{mn}}{R}r\right)sin(m\varphi)\exp(-ihz) \qquad \text{(CMW.12)}$$

$$\dot{H}_{\varphi}^{in} = -i\frac{\omega\varepsilon R}{\nu_{mn}}C_1 J_m'\left(\frac{\nu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \qquad \text{(CMW.13)}$$

$$\dot{H}_{z}^{in} = 0 \qquad \text{(CMW.14)}$$

$$h = h_{mn} = \sqrt{\omega^2\varepsilon\mu - \left(\frac{\nu_{mn}}{R}\right)^2} \qquad \text{(CMW.15)}$$

We write expressions for the reverse wave in the resonator, reflected from the wall of resonator, located in plane $z = l$. For this reason in the resulting expressions we change the sign before the value of a longitudinal constant $\dot{h}$ and we introduce the amplitude factor of the reflected wave C_3. (Fedorov, p. 224) As a result these replacements are obtained the expressions, which characterize the field of the reflected wave:

$$\dot{E}_{r}^{ref} = \frac{i\dot{h}R}{\nu_{mn}}C_3 J_m'\left(\frac{\nu_{mn}r}{R}\right)\cos(m\varphi)\exp(i\dot{h}z), \qquad \text{(CMR.1)}$$

$$\dot{E}_{\varphi}^{ref} = -\frac{i\dot{h}R^2}{\nu_{mn}^{2}r}C_3 J_m\left(\frac{\nu_{mn}r}{R}\right)\sin(m\varphi)\exp(i\dot{h}z), \qquad \text{(CMR.2)}$$

$$\dot{E}_{z}^{ref} = C_3 J_m\left(\frac{\nu_{mn}r}{R}\right)\cos(m\varphi)\exp(i\dot{h}z), \qquad \text{(CMR.3)}$$

$$\dot{H}_{r}^{ref} = -\frac{i\omega\varepsilon R^2}{\nu_{mn}^{2}r}C_3 J_m\left(\frac{\nu_{mn}r}{R}\right)\sin(m\varphi)\exp(i\dot{h}z), \qquad \text{(CMR.4)}$$

$$\dot{H}_{\varphi}^{ref} = -\frac{i\omega\varepsilon R}{\nu_{mn}} C_3 J'_m\left(\frac{\nu_{mn} r}{R}\right)\cos(m\varphi)\exp(i\dot{h}z), \qquad \text{(CMR.5)}$$

$$\dot{H}_z^{ref} = 0\,. \qquad \text{(CMR.6)}$$

Total field in the resonator is the superposition of incident (upper index "in") and reflected (upper index "ref") waves. The boundary conditions are:

$$\dot{E}_r^{\Sigma} = \dot{E}_r^{in} + \dot{E}_r^{ref} = 0 \text{ at } z = 0 \text{ and } z = l \qquad \text{(CMR.7)}$$

$$\dot{E}_{\varphi}^{\Sigma} = \dot{E}_{\varphi}^{in} + \dot{E}_{\varphi}^{ref} = 0 \text{ at } z = 0 \text{ and } z = l \qquad \text{(CMR.8)}$$

We substitute expressions of $\dot{E}_r^{in}$ (CMW.9), $\dot{E}_r^{ref}$ (CMR.1) and using boundary conditions with at $z = 0$, we will obtain:

$$i\frac{hR}{\nu_{mn}} J'_{mn}\left(\frac{\nu_{mn}}{R} r\right)\cos(m\varphi)(C_3 - C_1) = 0\,, \qquad \text{(CMR.9)}$$

It means that $C_3 = C_1$. (CMR.10)

Satisfying boundary conditions at $z = l$ and taking into account (CMR.10) we receive:

$$i\frac{hR}{\nu_{mn}} J'_{mn}\left(\frac{\nu_{mn}}{R} r\right)\cos(m\varphi) C_1\left(\exp(ihl) - \exp(-ihl)\right) = 0 \qquad \text{(CMR.11)}$$

[P.S. $\exp(-ihl) = 1/\exp(ihl)$]

Taking into account:

$$\frac{e^{ihz} - e^{-ihz}}{2i} = \sin(hz) \text{ and } \frac{e^{ihz} + e^{-ihz}}{2} = \cos(hz) \qquad \text{(CMR.12)}$$

we can write the boundary conditions (CMR.11) in the form:

$$-2\frac{hR}{\nu_{mn}} J'_{mn}\left(\frac{\nu_{mn}}{R} r\right)\cos(m\varphi) C_1 \sin(hl) = 0. \qquad \text{(CMR.13)}$$

On the basis this we obtain

$$\sin(hl) = 0 \text{ and } h = \frac{p\pi}{l}, \quad p = 0,1,2,3... \qquad \text{(CMR.14)}$$

Thus $h = \frac{p\pi}{l}, \quad p = 0,1,2,3...$ (CMR.15)

Using expressions (CMW.9-14) and (CMR.1-6) it is possible to write for the total field:

$$\dot{E}_r^{\Sigma} = -2\frac{p\pi R}{l\nu_{mn}} C_1 J'_m\left(\frac{\nu_{mn}}{R} r\right)\cos(m\varphi)\sin\left(\frac{p\pi}{l} z\right) \qquad \text{(CMR.16)}$$

$$\dot{E}_{\varphi}^{\Sigma}=2\frac{p\pi R^2}{l\nu_{mn}^2 r}C_1 J_m\left(\frac{\nu_{mn}}{R}r\right)\sin(m\varphi)\sin\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.17)}$$

$$\dot{E}_{z}^{\Sigma}=2C_1 J_m\left(\frac{\nu_{mn}}{R}r\right)\cos(m\varphi)\cos\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.18)}$$

$$H_{r}^{\Sigma}=-2i\frac{\omega\varepsilon R^2}{\nu_{mn}^2 r}C_1 m J_m\left(\frac{\nu_{mn}}{R}r\right)\sin(m\varphi)\sin\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.19)}$$

$$\dot{H}_{\varphi}^{\Sigma}=-2i\frac{\omega_{rez}\varepsilon R}{\nu_{mn}}C_1 J_m'\left(\frac{\nu_{mn}}{R}r\right)\cos(m\varphi)\cos\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.20)}$$

$$\dot{H}_{z}^{\Sigma}=0\,. \quad \text{(CMR.21)}$$

In the circular waveguide the electrical type waves designates E_{mnp} . Here index m is the number of the field variations along the φ coordinate and also the order of the Bessel function. n is the root of the Bessel function. Index p is the number of variations of the field along the axis Z of resonator. When than $p=0$ the components $\dot{E}_r^{\Sigma}$, $\dot{E}_{\varphi}^{\Sigma}$, disappear, but the components $\dot{E}_z^{\Sigma}, \dot{H}_{\varphi}^{\Sigma}, H_r^{\Sigma}$ which do not depend on the coordinate z (when z=0) remain.

EXPRESSIONS FOR THE FIELD COMPONENTS OF THE MAGNETIC FIELD IN A RESONATOR, CREATED ON THE BASE OF A CIRCULAR WAVEGUIDE.

(Fedorov ,p. 166)

$$k_{\perp mn}=\frac{\mu_{mn}}{R} \quad \text{(CMR.22)}$$

$$\dot{H}_{r}^{in}=-i\frac{hR}{\mu_{mn}}C_2 J_m'\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \quad \text{(CMR.23)}$$

$$\dot{H}_{\varphi}^{in}=i\frac{hR^2}{\mu_{mn}^2 r}C_2 m J_m\left(\frac{\mu_{mn}}{R}r\right)\sin(m\varphi)\exp(-ihz) \quad \text{(CMR.24)}$$

$$\dot{H}_{z}^{in}=C_2 J_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \quad \text{(CMR.25)}$$

$$\dot{E}_{r}^{in}=i\frac{\omega\mu R^2}{\mu_{mn}^2 r}C_2 m J_m\left(\frac{\mu_{mn}}{R}r\right)\sin(m\varphi)\exp(-ihz) \quad \text{(CMR.26)}$$

$$\dot{E}_{\varphi}^{in} = i\frac{\omega\mu R}{\mu_{mn}}C_2 J'_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(-ihz) \quad \text{(CMR.27)}$$

$$\dot{E}_{z}^{in} = 0 \quad \text{(CMR.28)}$$

$$h = h_{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\mu_{mn}}{R}\right)^2} \qquad (14.57) \quad \text{(CMR.29)}$$

For the reflected wave:

$$\dot{H}_{r}^{ref} = i\frac{hR}{\mu_{mn}}C_4 J'_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(ihz) \quad \text{(CMR.30)}$$

$$\dot{H}_{\varphi}^{ref} = -i\frac{hR^2}{\mu_{mn}^2 r}C_4 m J_m\left(\frac{\mu_{mn}}{R}r\right)\sin(m\varphi)\exp(ihz) \quad \text{(CMR.31)}$$

$$\dot{H}_{z}^{ref} = C_4 J_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(ihz) \quad \text{(CMR.32)}$$

$$\dot{E}_{r}^{ref} = i\frac{\omega\mu R^2}{\mu_{mn}^2 r}C_4 m J_m\left(\frac{\mu_{mn}}{R}r\right)\sin(m\varphi)\exp(ihz) \quad \text{(CMR.33)}$$

$$\dot{E}_{\varphi}^{ref} = i\frac{\omega\mu R}{\mu_{mn}}C_4 J'_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\exp(ihz) \quad \text{(CMR.34)}$$

$$\dot{E}_{z}^{in} = 0 \quad \text{(CMR.35)}$$

$$\omega_{rez} = c\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2} \qquad p = 1,2,3.. \quad \text{(CMR.36)}$$

$$c = \frac{1}{\sqrt{\varepsilon\mu}} \ , \ (\varepsilon = \varepsilon_0\varepsilon_r, \quad \mu = \mu_0\mu_r).$$

Superposition of the field of incident (forward) and reflected (reverse) waves, using boundary conditions $z = 0$, $z = l$ and taking into account formula (CMR.12) we receive:

$$C_4 = -C_2, \quad \text{(CMR.37)}$$

$$h = \frac{p\pi}{l}, \quad p = 1,2,3... \quad \text{(CMR.38)}$$

$$\dot{H}_{r}^{\Sigma} = -2i\frac{p\pi R}{l\,\mu_{mn}}C_2 J'_m\left(\frac{\mu_{mn}}{R}r\right)\cos(m\varphi)\cos\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.39)}$$

$$\dot{H}_{\varphi}^{\Sigma} = 2i\frac{p\pi R^2}{l\,\mu_{mn}^2 r}C_2 m J_m\left(\frac{\mu_{mn}}{R}r\right)\sin(m\varphi)\cos\left(\frac{p\pi}{l}z\right) \quad \text{(CMR.40)}$$

$$\dot{H}_z^{\Sigma} = -2iC_2 J_m\left(\frac{\mu_{mn}}{R} r\right)\cos(m\varphi)\sin\left(\frac{p\pi}{l} z\right) \quad \text{(CMR.41)}$$

$$\dot{E}_r^{\Sigma} = 2\frac{\omega_{rez}\mu R^2}{\mu_{mn}^2 r} C_2 m J_m\left(\frac{\mu_{mn}}{R} r\right)\sin(m\varphi)\sin\left(\frac{p\pi}{l} z\right) \quad \text{(CMR.42)}$$

$$\dot{E}_r^{\Sigma} = 2\frac{\omega_{rez}\mu R}{\mu_{mn}} C_2 J_m'\left(\frac{\mu_{mn}}{R} r\right)\cos(m\varphi)\sin\left(\frac{p\pi}{l} z\right) \quad \text{(CMR.43)}$$

$$\dot{E}_z^{ref} = 0 \quad \text{(CMR.44)}$$

In the circular waveguide the magnetic type waves designates E_{mnp}. Here index m and index p are the numbers of variations in the field along the φ and z coordinates. Index n is the number of the root of the derivative of Bessel function. A number for the index p begins from one, since with $p = 0$ all field components disappear.

DETERMINATION OF THE RESONANCE FREQUENCY 0F THE MAIN MODES FOR THE WAVES OF ELECTRICAL AND MAGNETIC TYPES IN THE CIRCULAR CYLINDRICAL RESONATORS.

The relationships for the longitudinal wave numbers in the case of the waves of electrical (CMW.15) and magnetic (CMR.28) types were brought out in the study of processes in the circular waveguide. These formulas are valid for the circular cylindrical cavity resonator with the difference that instead of the angular operating frequency $\omega = 2\pi f$ should be taken the resonance frequency ω_{rez} of the resonator. Furthermore, it is necessary to consider expression (CMR.15) in the case of electrical type waves. It is necessary to consider expression (CMR.38) in the case of magnetic type waves. The expressions (CMW.15) and (CMR.29) are written in the form.

$$\frac{p\pi}{l} = \sqrt{\omega_{rez}^2 \varepsilon\mu - \left(\frac{\nu_{mn}}{R}\right)^2} \quad \text{(CMR.46)}$$

and

$$\frac{p\pi}{l} = \sqrt{\omega_{rez}^2 \varepsilon\mu - \left(\frac{\mu_{mn}}{R}\right)^2} \quad \text{(CMR.47)}$$

From the expression (CMR.46) it is possible to obtain formula for the resonance frequency of circular cavity in the case of electrical type waves.

$$\omega_{rez} = \frac{1}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\nu_{mn}}{R}\right)^2} = c\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\nu_{mn}}{R}\right)^2} \quad (p = 0,1,2,3...) \qquad \text{(CMR.48)}$$

Then the resonance frequency f_{rez} and the resonance wavelength λ_{rez}

$$f_{rez} = \frac{c}{2\pi}\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\nu_{mn}}{R}\right)^2}, \quad p = 0,1,2,3... \qquad \text{(CMR.49)}$$

$$\lambda_{rez} = \frac{2\pi}{\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\nu_{mn}}{R}\right)^2}}, \quad p = 0,1,2,3... \qquad \text{(CMR.50)}$$

From the expression (CMR.47) it is possible to obtain formula for the resonance frequency of circular cavity in the case of magnetic type waves.

$$\omega_{rez} = \frac{1}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2} = c\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2}, \quad p = 1,2,3... \qquad \text{(CMR.51)}$$

$$f_{rez} = \frac{c}{2\pi}\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2}, \quad p = 1,2,3... \qquad \text{(CMR.52)}$$

$$\lambda_{rez} = \frac{2\pi}{\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2}}, \quad p = 1,2,3... \qquad \text{(CMR.53)}$$

Circular cavity is the multiwave system, for which basic mode has the maximum resonance wavelength.

With the intended sizes of resonator, as it follows from the relationships (CMR.50) and (CMR.53), fundamental waves appear with the smallest values of index p and roots of functions ν_{mn} and μ_{mn} (see next Tables Rez1, Rez2).

Table Rez1. Values of the roots ν_{mn} of the Bessel functions $J_m(\mathbb{Z})$, where $\mathbb{Z} = k_\perp r$

ции $J_m(gr)$.

Таблица 14.1

n	m = 0	m = 1	m = 2
1	2,405	3,832	5,135
2	5,520	7,016	8,417
3	8,654	10,174	11,620

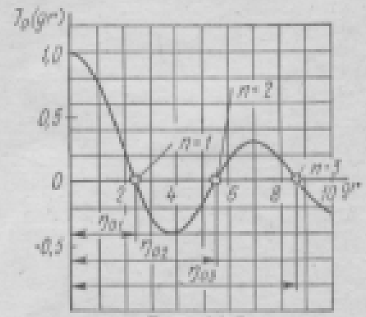

Рис. 14.5

Table Rez2. Values of the roots μ_{mn} of the derivatives of Bessel functions $\frac{dJ_m(\mathbb{Z})}{d\mathbb{Z}} = J'_m(\mathbb{Z})$, where $\mathbb{Z} = k_\perp r$

Таблица 14.2

n	m = 0	m = 1	m = 2
1	3,832	1,840	3,054
2	7,016	5,335	6,705
3	10,174	8,536	9,965

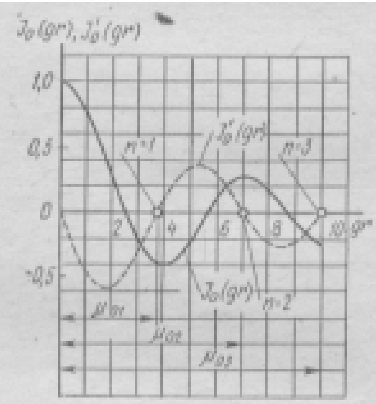

Рис. 14.6

From tables Rez1, Rez2 we see that the smallest value of the roots are $\nu_{01} = 2.405$ and $\mu_{11} = 1.84$. In the case of electrical type waves is permissible the value of $p = 0$. Consequently, the basic oscillation of the electrical type is E_{010} with the resonance frequency:

$$\omega_{rez(E_{010})} = c\frac{2.405}{R}. \qquad \text{(CMR.54)}$$

The basic oscillation of the electrical type is H_{111} with the resonance frequency :

$$\omega_{rez(H_{111})} = c\sqrt{\left(\frac{\pi}{l}\right)^2 + \left(\frac{1.84}{R}\right)^2} \qquad \text{(CMR.55)}$$

EXISTENCE CONDITIONS FOR OF ASSIGNED TYPE WAVES IN THE RESONATOR.

The existence conditions of assigned type waves are determined, first of all, by the possibility of their propagation in the waveguide, on base of which is created the resonator. For electrical type waves propagation condition they are inequality $\lambda \prec \lambda_{cut}$, where

$$\lambda_{cut(E_{mn})} = \frac{2\pi R}{\nu_{mn}} = \frac{6.28R}{\nu_{mn}}$$

For magnetic type waves propagation condition

$$\lambda_{cut(H_{mn})} = \frac{2\pi R}{\mu_{mn}} = \frac{6.28R}{\mu_{mn}}$$

In the resonator the wavelength of fluctuation is equal to resonance wavelength, determined by relationships (CMR.50) and (CMR.53). Accordingly propagation conditions are written in the form:

$$\frac{2\pi}{\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\nu_{mn}}{R}\right)^2}} \prec \frac{2\pi R}{\nu_{mn}} \qquad \text{(CMR.56)}$$

$$\frac{2\pi}{\sqrt{\left(\frac{p\pi}{l}\right)^2 + \left(\frac{\mu_{mn}}{R}\right)^2}} \prec \frac{2\pi R}{\mu_{mn}} \qquad \text{(CMR.57)}$$

These inequalities are always fulfilled when $p \neq 0$. In the case of electrical type waves at the index $p = 0$ the inequality (CMR.56) passes into the equality. In the resonator in this case ($p = 0$) there are oscillations, which correspond to critical case, when are fulfill the boundary conditions for the propagation of modes E_{mn} in the circular waveguide.

PICTURES OF THE EM FIELDS DISTRIBUTIONS IN THE CIRCULAR CAVITY REZONATOR.

Comparing expressions of the EM field components in a waveguide (CMR.1-6) and the EM field components in a resonator (CMR.16-21), it is possible to establish that the nature of EM field change along the coordinates r and φ in the resonator of and in the waveguide is the same.

The satisfaction of boundary conditions in the resonator on the end faces leads to the fact that electrical and magnetic fields in the resonator have displacement relative to each other to quarter wavelength in the comparison with their position in the waveguide.

Therefore it is not difficult to construct the picture of field in the cavity resonator on the known picture of field in the waveguide with the same indices m and n.

Further are given some pictures of the electromagnetic field distributions.